Forage Res., **39** (1) : pp. 48-51 (2013) **http://forageresearch.in**

SOME BALANCED BIPARTITE ROW-COLUMN DESIGNS

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(Received : 24 August 2013; Accepted : 26 September 2013)

SUMMARY

A series of balanced bipartite row-column designs (BBPRC-designs) into sets of treatments with v_1=4t+3 (test treatments) v_2=2t+2 (control treatments) with varying replication has been obtained by supplementing a set of v_2=2t+2 treatments in blank positions of non-orthogonal RC design of Agrawal (1966). Such designs find applications in agricultural and industrial experimentation where the investigator wishes to provide unequal replications to two sets of treatments or wishes to estimate the two sets with different precaution and at the same time wants to estimate heterogeneity due to two sources. The variances for different types of comparison have also been worked out.

Key words : BBPRC design, general efficiency balanced designs, non-orthogonal row-column designs

In many situations in agriculture, forestry, animal science and plant development programme the investigator wants to provide unequal replications to two sets of treatments comparison of one set of V_1 test treatments and second set v_2 control treatments. Likewise, in bioassays experimentations. The preparation contrast and the combined regression contrast being the most important need to be estimated with higher efficiency as compared to the other contrasts. Some time depending upon the natural heterogeneity due to two sources needs to be estimated. The balanced bipartite row-column designs constructed in this paper are most appropriate under such situations. Hanusz (1995) has applied RC-design in the study of relative policy of two preparations estimating two sources of variations. Like age and weight of the subjects as rows and columns.

PRELIMINARIES AND NOTATIONS

Consider a Row-column Design (RC-design) D (v, p, q, r) in v treatments allotted to n=pq units arranged in p rows and q columns such that each unit receives exactly one treatment and i-th treatment is replicated r_i(i=1,2,……………. v) times. Let N_1 and N_2 denote treatments verses rows and treatments verses columns incidence matrices of the orders v x p and v x q, respectively, of the RC-design D.

Let R=diag($r_1,r_2,\ldots\ldots r_v$) be v x v diagonal matrix and

r=($r_1,r_2\ldots\ldots r_v$) be v x 1 replication vector of the RC-design D.

Then the Coefficient Matrix C of the reduced Normal equations for estimating treatments parameter (C t =Q) is given by

$$C=R-N_1N_1'/q-N_2N_2'/p+rr'/pq \qquad (2.1)$$

In particular if $N_1=N_2$ and p=q hold then C-matrix further simplifies to

$$C= (1/p^2)\ [p^2R-2p\ N_1\ N_1' +rr'] \qquad (2.2)$$

In most of the practical situations in agricultural experiments we want to compare two disjoint sets of treatments says 1st set of v_1 treatments called "test treatments" each replicated r_1 times and 2nd set of v_2 treatments called "control treatments" each replicated r_2 times.

Then the replication vector r, the diagonal matrix R and the incidence matrices N_1 and N_2 for such RC-designs are of the forms

$$r=[r_1 1v_1,' r_2 1v_2]',\ R=\text{diag}\ [r_1^{\delta},\ r_1^{\delta}]$$
$$N_1=[N_{11}\quad N_{12}]'\ ;\ N_2=[N_{21}\ N_{22}] \qquad (2.3)$$

Where, N_{1i},N_{2i} (i=1,2) are the treatments verses rows and treatments verses columns incidents matrices of orders v_i x p and v_i x q for v_1 test treatments and v_2 control treatments, respectively.

The C-matrix of such RC-designs at (2.1) can be put in the form

$$C = \begin{matrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{matrix} \qquad (2.4)$$

here C_{ij} (i. j=1,2) are the matrices of orders v_i x v_1. Parsad and Gupta (2001) have shown that if the C-matrix of the RC-design is of the form

$$C = \frac{1}{pq} \begin{matrix} v1s1 + v2s0)Iv1\text{-}s1Jv1 & \text{-}s0Jv1xv2 \\ \text{-}s0Jv2xv1 & (v2s2+v1s0)Iv2\text{-}s2Jv2 \end{matrix} \qquad (2.5)$$

Where, s_0, s_1 and s_2 are constants, then the RC-design is Balanced bipartite Row-Columns design (BBPRC-design) with parameters v_1, v_2, r_1, r_2, p, q, s_0, s_1, s_2.
Further if $s_o^2 = s_1 s_2$ holds then the designs are called General efficiency balanced row-column designs (GEB RC designs) of Gupta and Gandhi Prasad (1990).
The variances for three types of comparisons (for BBPRC-designs can be obtained as $v(t_i\text{-}t_i)=2pqs^2/v_1s_1+v_2s_0)$
if t_i & t_j ΣIst group of v_1 treatments
$= 2pqs^2/v_2s_2+v_1s_0)$

t_i & t_j ε IIst group of v_2 treatments

$$=pq\sigma^2[\frac{v2so+s1}{v2so(v1s1+v2so)}+\frac{v2-1}{v2(v2s2+v1so}$$

Or

t_i ε Ist group of v_2 treatments and

$$=pqs^2\ [\ \frac{vIso+sI}{v1so(v2s2+v1so)}+\frac{v1-1}{vI(vIsI+v2so}$$

t_j ε IIst group of v_2 treatments
σ^2 being the variance of the design (2.6)

Lot of work on construction and analysis of Bipartite Block design and GEB block design has been done by Das and Gosh (1985), Kageyama and Mukherjee (1986), Kageyama and Sinha (1988) and Gupta and Prasad (2001). But very little work has been done on construction and properties of Balanced bipartite row-column designs by Prasad and Gupta (2001) and on GEB RC-designs by Gupta ad Ghandhi Prasad (1990). Parsad *et al*. (2003) used BBPRC-designs for the construction of structurally incomplete Row-columns designs (SIRC-designs) and Balanced Bipartite Block designs (BBPB-designs). Hanusz (1995) applied RC-designs in the study of relative potency of two preparations and eliminating two sources of variations like ages and weights of subjects as rows and columns. In this paper a series of balanced bipartite row-column designs has been constructed, which for different values of v provides BBPRC designs with different parameters.

METHOD OF CONSTRUCTION AND ANALYSIS

Consider the SBIB/SBIB Rc-design D of Agrawal (1966) with parameters

$$v=p=q=4t+3,\ r=k=h=2t+1 \qquad (3.1)$$

and having $N_1=N_2=W=N$ say, herer N is the incidence matrix of the symmetrical BIB with
parameters

$$v=b=\ 4t+3, \qquad r=k=2t+1,\ \gamma=1$$

Agrawal (1966) discussed the procedure for construction of such RC-designs. For the sake of convenience RC-designs are expressed in notations of Zeelan and Feeder (1964) as (Column Incidence matrix N_2)/(Row Incidence matrix N_1). Thus, the above RC-design considered in this series is SBIB/SBIB RC-design.

Let 1,2,…………..4t+3 denote the treatments of the basic design. Fill up the remaining (2t+2) positions in the first row with v_2=2t+2 treatments, namely, 1α, 2α .., (2t+2) α. The remaining rows are to be filled up diagonally cyclically with the same elements. Thus, each of the new treatments will appear once in each row and once in each column. Then we prove the following.

Theorem : The design D* is BBP RC-design with parameters

v_1*=4t+3,v_2*=2t+2,r_1*=2t+1,r_2*=4t+3,s_0=(2t+1)/(4t+3), s_1=(4t²+2t-1)/(4t+3)2 s_2=1

Proof : The parameters v_1*, v_2*,p*,q*,r_1*,r_2* are obvious and need no explanation. The treatment verses row and treatments verses columns incidence matrices for the design D_3* are :

$$N_1{*}=N_2{*}= \frac{N}{Jv_2\ xv\ 1}$$

$N_1{}^*N_1{}^{*'}=N_2{}^*N_2{}^{*'}=$

$$\begin{array}{ll} (t+1)Iv_1+tJv\ 1 & (2t+1)Jv_1\ xv_2 \\ (2t+1)Jv_2\ xv1 & (4t+3)Jv_2 \end{array}$$

and

$$r^*=[(2t+1)Iv_1(4t+3)tv_2]$$

implies

$r^*r^{*'}=$

$$\begin{array}{ll} (2t+1)Jv_1 & (2t+1)(4t+3)Jv_1xv_2 \\ (2t+1)4t+3)\ Jv_2xv_1 & (4t+3)2Jv_2 \end{array}$$

and

$R^*=$

$$\begin{array}{ll} (2t+1)Jv_1 & 0v_1xv_2 \\ 0v_2xv_1 & (4t+3)^2Iv_2 \end{array}$$

Since for the RC design D*, P*=q* and $N_1{}^*= N_2{}^*$ holds, so the C matrix of RC-design D* is obtained using (2.2) and can be put in the form (2.4) on simplification as

$C^*=$

$$\begin{array}{ll} 1/(4t+3)^2[\{(4t+3)(8t^2+8t+1)-(4t^2+2t-1 & -(2t+1)\ Jv_1xv_2\ /(4t+3) \\ +(4t^2+2t-1)\}Iv_1-(4t^2+2t-1)Jv_1] & \\ -(2t+1)Jv_2xv_1\ /(4t+3) & [(4t+2+1)Iv_2-Jv_2] \end{array}$$

Hence, the resultants design D* is BBPRC-design with parameters (3.2)

Illustration : Consider the following SBIB/SBIB RC-design D of Agrawal (1966) with parameters V=p=q=7, h-k=3, r=3

	1	2	3	4	5	6	7
1		3	5		2		
2			4	6		3	
3				5	7		4
4	5				6	1	
5		6				7	2
6	3		7				1
7	2	4		1			

Fill up the four positions of first row by treatments 1α, 2α, 3α and 4α and fill up the remaining rows diagonally as explained earlier we get the following design.

	1	2	3	4	5	6	7
1	1α	3	5	2α	2	3α	4α
2	4α	1α	4	6	2α	3	3α
3	3α	4α	1α	5	7	2α	4
4	5	3α	4α	1α	6	1	2α
5	2α	6	3α	4α	1α	7	2
6	3	2α	7	3α	4α	1α	1
7	2	4	2α	1	3α	4α	1α

The above design is BBP RC-design has the parameters v_1=7, v_2=4, p*=q*=7, $r_1{}^*$=3, $r_2{}^*$=7, s_0=3/7, s_1=5/49, s_2=7

APPLICATION

Besides in agriculture Bipartite row-column designs have wide applications in medical science, animal science and industrial experimentations. In field experiments rows and columns may be considered as two perpendicular directions of fertility variation. Likewise in animal science for studying the milk yield rows and columns may be taken as lactation numbers and breeds. These designs are advantageous over latin square as numbers of breeds, feeds and lactations all may be different and need smaller number of animals to test the same number; of feeds as compared to that in latin square design and at the same time eliminating heterogeneity due to two sources thus resulting in higher precision.

Freeman (1975) while studying the effect of chemicals to control the plant diseases considered columns as residual effect of treatments of the previous trail in RBD and the rows as blocks of RBD experiment currently under study. Henusz (1995) while studying the relative potency of two preparations, namely, standard and test preparations of drugs, herbicides and pesticides, etc. considered ages and weights of subjects as rows and columns, respectively. Parsad *et al.* (2003) suggested the use of Bipartite rows and columns designs for testing the breaking strength of several synthetic fibers divided in two sets of treatments, while rows and columns may be machines and operators/shifts. Parsad *et al.* (2003) also suggested the use of these designs in experiments where experimental units are long lived like perennial crops or animals or in crop sequence where two sets of treatments are applied in succession.

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