LATIN SQUARE TYPE BIPARTITE ROW-COLUMN DESIGNS

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SUMMARY

A class of Balanced Bipartite Row-Column Designs (BBPRC-designs) in two sets of treatments of sizes v_1=v and v_2=v+1 with replications 2(v-1) and 2v, respectively, has been constructed by replacing the ii-th positions of 2v × 2v standard cyclic latin square design in 2v treatments by 2v treatments by (2v+1)-th treatment. Such design finds applications in agricultural and industrial experimentations where different replications for two sets of treatments are to be provided or we wish to estimate the two sets of treatments with different precisions and at the same time want to eliminate two way heterogeneity.

Key words : Standard cyclic latin designs, balanced bipartite row-columns design, GEB-designs

One of the principal problems in plant breeding and in biochemical research of new pesticides, soil fumigants, drugs, etc. is the evaluation of new strains or chemicals (test treatments). There are certain situations where the material for new strains or chemicals for the test treatments is so small that it is sufficient for making one or two observations only, but there may not be any such limitations for standard varieties (control treatments). Efficient experimental design and efficient procedure are necessary in these circumstances in order to make best use of available resources. Likewise in bioassay experimentations the preparation contrast and the combined regression contrasts being the most important, need to be estimated with higher efficiency as compared to the other contrasts. Hence, the investigator needs an 'ad hoc' design which can provide comparison of a set of v_1 "test treatments" each replicated r_1 times with another set of v_2 "control treatments" each replicated r_2 times and at the same time eliminate two way heterogeneity. Balanced bipartite row-columns designs (BBPRC-design) and General efficiency balanced row-columns designs (GEBRC-designs) can be appropriately used under above mentioned circumstances.

PRELIMINARIES AND NOTATIONS

Consider a Row-column Design (RC-design) D (v, p, q, r) in v treatments allotted to n=pq units arranged in p rows and q columns such that each unit receives exactly one treatment and i-th treatment is replicated r_i(i=1,2,……………. v) times. Let N_1 and N_2 denote treatments verses rows and treatments verses columns incidence matrices of the orders v x p and v x q respectively of the RC-design D.

Let R=diag(r_1,r_2……….r_v) be v x v diagonal matrix and

r=(r_1,r_2………. r_v) be v x 1 replication vector of the RC-design D.

Then the Coefficient Matrix C of the reduced Normal equations for estimating treatments parameter (C t =Q) is given by

$$C = R-N_1N_1'/q-N_2N_2'/p+rr'/pq \qquad (2.1)$$

In particular if N_1=N_2 and p=q hold then C-matrix further simplifies to

$$C= (1/p^2) [p^2R-2p\ N_1\ N_1' +rr'] \qquad (2.2)$$

In most of the practical situations in agricultural experiments we want to compare two disjoint sets of treatments says 1st set of v_1 treatments called "test treatments" each replicated r_1 times and 2nd set of v_2 treatments called "control treatments" each replicated r_2 times.

Then the replication vector r, the diagonal matrix R and the incidence matrices N_1 and N_2 for such RC-designs are of the forms

$$r=[r_1 1v_1,' r_2 1v_2]', \quad R=diag\ [r_1^{\delta},\ r_1^{\delta}]$$

$$N_1 = [N_{11}\quad N_{12}]'\ ;\ N_2 = [N_{21}\ N_{22}],' \qquad (2.3)$$

Where, N_{1i},N_{2i} (i=1,2) are the treatments verses rows and treatments verses columns incidents matrices of orders v_i x p and v_i x q for v_1 test treatments and v_2 control treatments, respectively.

The C-matrix of such RC-designs at (2.1) can be put in the form

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \quad (2.4)$$

here C_{ij} (i. j=1,2) are the matrices of orders v_i x v_1. Parsad and Gupta (2001) have shown that if the C-matrix of the RC-design is of the form

$$C= \frac{1}{pq} \begin{bmatrix} v1s1+v2s0)Iv1-s1Jv1 & -s0Jv1xv2 \\ -s0Jv2xv1 & (v2s2+v1s0)\ Iv2-s2Jv2 \end{bmatrix} \quad (2.5)$$

Where, s_0, s_1 and s_2 are constants, then the RC-design is Balanced bipartite Row-Columns design (BBPRC-design) with parameters v_1, v_2, r_1, r_2, p, q, s_0, s_1, s_2.

Further if $s_o^2=s_1s_2$ holds then the designs are called General efficiency balanced row-column designs (GEB RC designs) of Gupta and Gandhi Prasad (1990).

The variances for three types of comparisons (for BBPRC – designs can be obtained as

$v(t_i-t_i)=2pqs^2/v_1s_1+v_2s_0)$

if t_i & t_j[* Ist group of v_1 treatments

$= 2pqs^2/v_2s_2+v_1s_0)$

t_i & t_j $\in$ IIst group of v_2 treatments

$=pq\sigma^2$ [

Or

t_i $\in$ Ist group of v_2 treatments and

$=pq\sigma^2$ [

t_j $\in$ IIst group of v_2 treatments

σ^2 being the variance of the design (2.6)

Lot of work on construction and analysis of Bipartite Block design and GEB block design has been done by Das and Gosh (1985), Kageyama and Mukherjee (1986), Kageyama and Sinha (1988) and Gupta and Prasad (2001). But very little work has been done on construction and properties of Balanced bipartite row-column designs by Prasad and Gupta (2001) and on GEB RC-designs by Gupta ad Ghandhi Prasad (1990). Parsad *et al*. (2003) used BBPRC-designs for the construction of structurally incomplete Row-columns designs (SIRC-designs) and Balanced Bipartite Block designs (BBPB-designs). Hanusz (1995) applied RC-designs in the study of relative potency of two preparations and eliminating two sources of variations like ages and weights of subjects as rows and columns. In this paper, a series of balanced bipartite row-column designs has been constructed, which for different values of v provides BBPRC designs with different parameters.

METHOD OF CONSTRUCTION AND ANALYSIS

Let D be the orthogonal RC-design in the form of 2v x 2v cyclic standard latin square. Such design can always be constructed for all values of v by taking 1st row (1st column) in alphabetic order developing remaining rows (columns) cyclically.

The RC-design D has the following parameters

$$v' = 2v=p=q, \quad r =2v, \quad n=4v^2 \quad (3.1)$$

The RC-design D so constructed always has treatments 1,3,5……………., 2v-1 in the diagonal positions. Replace all the diagonal treatments by the treatments number '2v+1' and renumber the odd and even treatment as

$$\begin{bmatrix} 2[Iv+(v-2)Jv] & 2(v-1)Jvx(v+1) \\ 2(v-1)J(v+1)xv & 2vJv+1 \end{bmatrix}$$

2v- 1 → v

v-1, 2v → 2v

n we prove the following :

Theorem : The design D* is Balanced Bipartite Row-Columns design (BBPRC-design) with parameters

$v_1^* = v$, $v_2^* = v+1 = p^* = 2v$, $r_1^* = 2(v-1)$, $r_2^* = 2v$

$n^* = 4v^2$, $s_0 = 4v(v-1)$ $s_1 = 4(v^2 -2v -1)$, $s_2 = 4v^2$

Proof : The parameters v_1*, v_2*, p*,q*,r_1*,r_2* and n* are obvious and need no explanation, while the treatments verses rows and the treatments verses columns incidence matrices are :

$$N_1^* = N_2^* = 1_2 \text{ ' } \frac{Jv - Iv}{J(v+1)xv}$$

$N_1^*N_1^{*\prime}=N_2^*N_2^{*\prime}=$

and r* = [2(v-1)1'v,2v1'v+1]' implies

r*r*=

and R* = diag3.2 [2(v-1) Iv, 2v Iv+1]
Since for the RC design D*,
we have N1* = N2*, p* =q* holds
So the C matrix of the RC-design D* is obtained using (2.2) and be put in the form (2.4) on simplification as
C*=1/4v²

$$\begin{bmatrix} [\{4v(v^2-2v-1)+(v+1)x4v(v-1)Iv & [4v(v-1)Jvx(v+1)-4(v^2-2v-1)Jv] \\ 4v(v-1)J(v+1)xv & [(v+1)4v^2+4v^2(v-1)]I_{v+1}-4v^2J_{v+1} \end{bmatrix}$$

Comparison with (2.5) implies that D* is Balanced Biparitite Row-Column design (BBPRC-design) with parameters (3.2)
Hence the proof.
The variances for different types of comparisons V (t_i-t_j) can be easily obtained using (2.6)

Illustration : For V=4 consider RC-design D as 8 x 8 standard cyclic latin square with parameters
V'=2 v =p = q = 8, r = 8, n = 64

1	2	3	4	5	6	7	8
2	**3**	4	5	6	7	8	1
3	4	**5**	6	7	8	1	2
4	5	6	**7**	8	1	2	3
5	6	7	8	**1**	2	3	4
6	7	8	1	2	**3**	4	5
7	8	1	2	3	4	**5**	6
8	1	2	3	4	5	6	**7**

Replacing the diagonal contents (1,3,5,7) by treatment unmber '9' and renumber the odd and even treatments as :
1→1, 3→2, 5→3, 7→4, 2→5, 4→6, 6→7, 8→8
We get the following BBPRC design D*

9	5	2	6	3	7	4	8
5	**9**	6	3	7	4	8	1
2	6	**9**	7	4	8	1	5
6	3	7	**9**	8	1	5	2
3	7	4	8	**9**	5	2	6
7	4	8	1	5	**9**	6	3
4	8	1	5	2	6	**9**	7
8	1	5	2	6	3	7	**9**

With parameters $v_{1=}4$, $v_{2=}5$, p* = p* = 8, r_1* = 6, r_2=8, n*=64, $s_{0=}48$, $s_{1=}28$, $s_{2=}64$
Some useful BBPRC designs in two sets of treatment along with their replications and other parameters obtainable from the above series are listed in Table 1.

APPLICATIONS

Besides in agriculture Bipartite row-column designs have wide applications in medical science, animal science and industrial experimentations. In field experiments rows and columns may be considered as two perpendicular directions of fertility variation. Likewise in animal science for studying the milk yield rows and columns may be taken as lactation numbers and breeds. These designs are advantageous over latin square as numbers of breeds, feeds and lactation all may be different and need smaller number of animals to test the same number of feeds as compared to that in latin square design and at the same time eliminating heterogeneity due to two sources thus resulting in higher precision.

TABLE 1

v	v_1=v	v_2=v+1	r_1=2(v-1)	r_2 = 2v	p = 2v	q = 2v
2	2	3	2	4	4	4
3	3	4	4	6	6	6
4	4	5	6	8	8	8
5	5	6	8	10	10	10
6	6	7	10	12	12	12
7	7	8	12	14	14	14
8	8	9	14	16	16	16
9	9	10	16	18	18	18
10	10	11	18	20	20	20
11	11	12	20	22	22	22

Freeman (1975) while studying the effect of chemicals to control the plant diseases considered columns as residual effect of treatments of the previous trail in RBD and the rows as blocks of RBD experiment currently under study. Henusz (1995) while studying the relative potency of two preparations, namely, standard and test preparations of drugs, herbicides and pesticides etc. considered ages and weights of subjects as rows and columns respectively. Parsad *et al*. (2003) suggested the use of Bipartite rows and columns designs for testing the breaking strength of several synthetic fibers divided in two sets of treatments, while rows and columns may be machines and operators/shifts. Parsad *et al.* (2003) also suggested the use of these designs in experiments where experimental units are long lived like perennial crops or animals or in crop sequence where two sets of treatments are applied in succession.

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